Plasma Sheath Transducer for Axisymmetric Re-Entry Vehicles

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For aerodynamic stability, the center of gravity must be forward of the center of pressure Frequently, conical and other axisymmetric re-entry vehicles require high-density ballast (e g , lead) located as far forward as possible to achieve stability Ballast preempts useful payload This note describes a transducer that serves two functions: it substitutes for ballast, and it provides useful information concerning the plasma sheath

The transducer described herein is similar to one developed for measurement of electrical conductivity-velocity σU in arc plasmajets and rocket exhausts [1] It is an extension of a transducer described by Shercliff[2] and by Lehde and Lang [3] It consists of three circular coils with a common axis The two outer coils are driven at audio frequencies and are connected so that the magnetic fields are opposing Between the primary coils is a sensing coil The voltage that appears at the terminals of the sensing coil can be related to σU of the flow

The transducer will be located at the forward tip of the re-entry vehicle, if it is to serve as ballast In the stagnation region and just beyond the sonic line, the flow is changing rapidly The question is, how is the sensing coil voltage related to the flow properties? This note answers this question and briefly describes an experiment conducted in an arc plasmajet

A re-entry vehicle with ablation cooling is considered The tip of the vehicle requires a thick layer of ablation material as shown in Fig 1 The coil arrangement is also shown in Fig 1 Calculations were made for the geometry illustrated

The emf induced in the sensing coil is

$$e = -\frac{N\mu_0\omega}{4\pi}\int_A\int_V \frac{\mathbf{J}\times\mathbf{R}}{R^3}\,\mathbf{n}dAdV \qquad (1)$$

where J is current density in the plasma sheath, R the distance from JdV to an element of the sensing coil dA with normal n, N the number of turns in the sensing coil, ω the frequency of the primary magnetic field, and μ_0 the magnetic permeability J arises from the interaction of the applied magnetic field and the flowing plasma The electrical field is zero as a result of the axisymmetric geometry and the nonconducting surface

It is convenient, experimentally, to measure a response function that is closely related to mutual inductance The response function equals the voltage induced in the sensing coil due to a current ring of radius r located at z The current density in the ring is unity Experimentally, a traversing mechanism varies z, and different coils give variations in r

One obtains the response function mathematically by replacing J in Eq (1) by $e_\theta\delta(r,z)$ The result is

$$f(r,z) = -\frac{\mu_0 N\omega}{4\pi}\int_A \frac{\mathbf{e}_\theta\times\mathbf{R}}{R^3}\,\mathbf{n}dA \qquad (2)$$

In terms of the response function, the signal is

$$e = \int_V J(r,z)f(r,z)dV = J^*\int_V \frac{J}{J^*}f\,dV \qquad (3)$$

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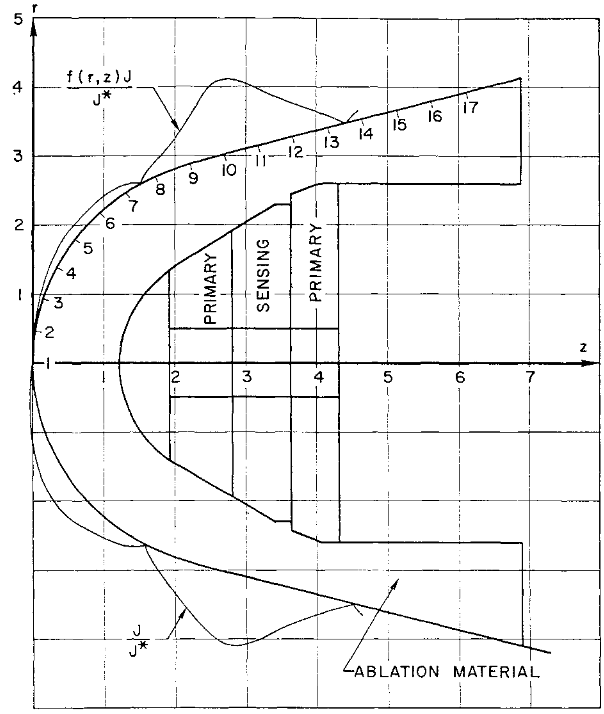

Fig 1 Coil arrangement for the σU instrument

where the volume integration extends to the regions having currents J The current density ratio J/J^* is defined in Eq (4)

To determine J, which results from the flow-magnetic field interaction, one must know σ, U, and B Both σ and U were determined from data (based on inviscid flow-field calculations) given in Ref 4 B was measured by means of a small coil that was traversed through the alternating primary field Components B_r and B_z were measured as a function of r and z

A current density ratio is defined as

$$\frac{J}{J^*} = \frac{\sigma}{\sigma^*}\frac{U_t}{U_t^*}\frac{B_n}{B_n^*} \qquad (4)$$

where the asterisk values occur at the intersection of the surface of the ablating material (preflight thickness) and the midplane of the sensing coil The subscript t denotes the component tangential to the surface of the nose, and n the normal component Interpretation of Eq (4) is as follows: if flow conditions exist so as to produce current density J^* at the sensing coil, then a current density J occurs at r and z The ratio J/J^* indicates the magnitude of a current ring at r and z compared to current density at the sensing coil Curves of measured B_n/B_n^* are shown in Fig 2 The velocity ratio U_t/U_t^*, as obtained from O'Brien,[4] also is plotted in Fig 2 These are the values of U at the surface of the nose

Reference 4 gives the temperature of the gas at the body A plot of T/T^* is given in Fig 3 It was assumed that $\sigma/\sigma^* = (T/T^*)^{1/2}$ Calculations based on this assumption are plotted in Fig 3

To illustrate the calculation and to indicate the interpretation for a signal voltage, one can calculate the relative contributions to the signal voltage e for a thin layer of the plasma sheath next to the body The product of the response function and the current density ratio J/J^* indicates the spatial origin of the signal e A plot of fJ/J^* appears in Fig 1 Most of the signal is due to that portion of the flow centered on the sensing coil In this region U and T are not changing rapidly Hence, average values, as measured by the instrument, and local values will be nearly the same To obtain the voltage at terminals of the sensing coil, an integration is

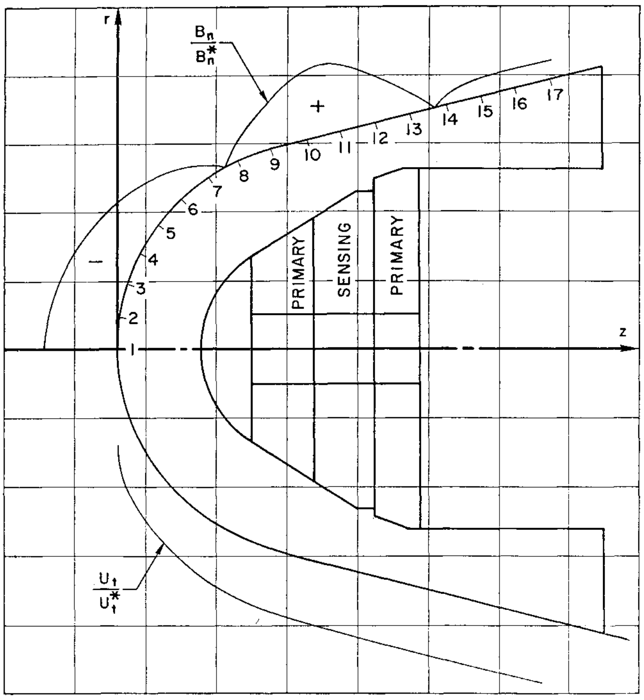

Fig 2 Plot of measured normal component of magnetic field and calculated tangential component of velocity

necessary A value of $J^* = \sigma^* U_t^* B_n^*$ can be estimated Hence,

$$e = \sigma^* U_t^* B_n^* \int_V \frac{fJ}{J^*} dV \tag{5}$$

For the case being considered here, the volume element is $2\pi r\ tds$, where r_s is radius of vehicle at z, t the thickness of the plasma layer, and ds distance along the nose contour

To extend the calculations to include the complete region between the shock wave and the body, one needs only to integrate Eq (3) throughout this volume Sufficient data have been obtained to evaluate $f(r,z)$; however, there was

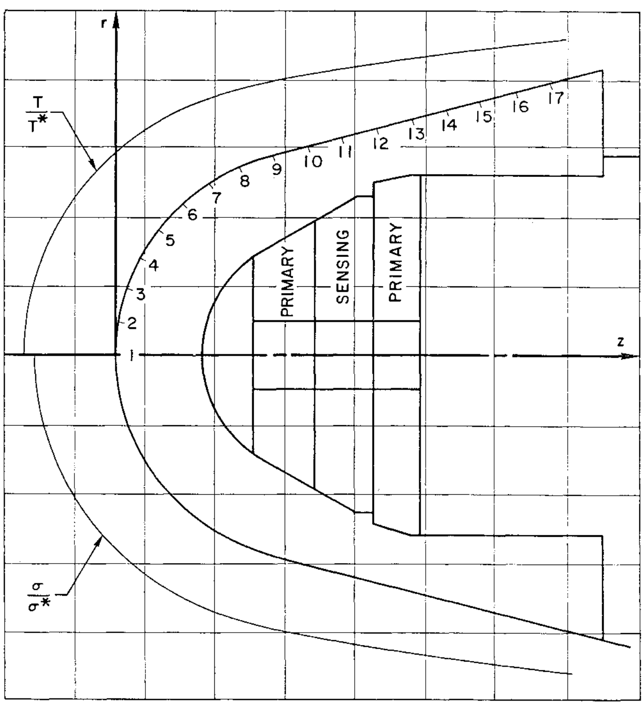

Fig 3 Plot of temperature ratio and electrical conductivity ratio (Values are at surface of nose)

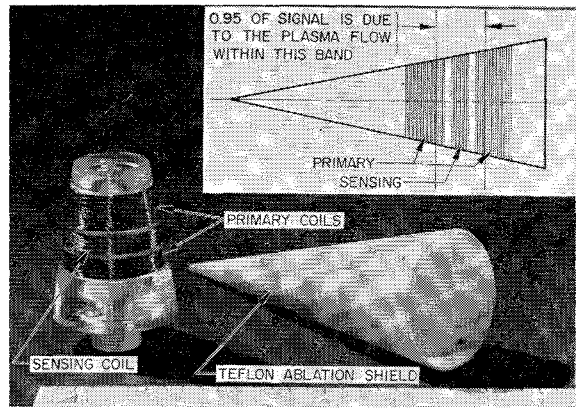

Fig 4 Coil geometry and Teflon protective shield used in arcjet experiments

not sufficient information available concerning the flow field to evaluate $J(r,z)$ outside a thin layer next to body

Description of a Plasma Arcjet Experiment

The aim of this experiment was to demonstrate that a signal could be obtained with the axisymmetric transducer immersed in a conducting gas The transducer was made small to insure that it would be immersed in the plasmajet and that the circular symmetry of the flow would not be distorted The base of the cone, as can be seen in Fig 4, was 2 in long Figure 5 illustrates a Teflon cone and coil assembly after a test Note the blunting of the nose tip

The response function $f(r,z)$ was not measured for this geometry By use of the formulas of Grover[5] it was possible to calculate $f(r,z)$ The primary magnetic field was measured To evaluate Eq (5) it was assumed that σ and U did not vary along the cone; with this assumption J/J^* equals B_n/B_n^* It was further assumed that $\sigma^* U^*$ equalled the freestream value With these assumptions the calculated signal for one test was 280 mv The observed signal was 21 mv One reason why the observed signal was low may be the influence of Teflon ablation products Also, the plasma properties in the jet were estimated using an energy balance with thermodynamic equilibrium and isentropic expansion assumed

There is a definite phase angle between the current in the primary coils and the signal The observed phase angles were in agreement ($\pm 5°$) with the predicted value

This note has shown that most of the signal comes from a narrow region of flow near the sensing coil Two reasons account for this: first, the response function falls off sharply as the current rings move away from the sensing coil; second, the normal component of the primary magnetic field, which drives the currents, falls off rapidly away from the sensing coil In the case of the transducer near the tip of the reentry vehicle, there is a third reason: the value of U is small, and the small U is partially offset by the increased σ

The arcjet experiments demonstrated that, in fact, signals could be obtained For the test conditions, the signal-to-noise ratio was large; succinctly stated, there was a strong

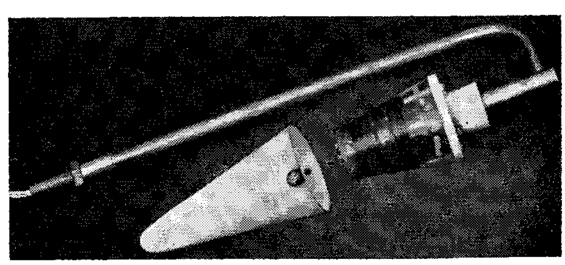

Fig 5 Teflon cone and coil assembly after a test

signal The difference between the observed and calculated values were probably due to errors in predicting $\sigma^* U^*$ The aim of the experiment was to demonstrate that the instrument responds to the plasma flow; this was demonstrated

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Laminar Boundary-Layer Development on Yawed Cone

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If an integral method[1] is used in investigating the supersonic three-dimensional boundary layer on a yawed cone, the boundary-layer development in the direction along the cone generator is needed Because of the conical configuration, Blasius-type parabolic similarity may be assumed[2] to exist in that direction, giving the following expression as the boundary-layer thickness:

$$\Delta = K(\mu\ x/\rho\ u\ )^{1/2} \tag{1}$$

where

Δ = boundary-layer thickness in terms of transformed coordinate Y

$Y = \int \frac{\rho}{\rho_e}\, dy$

x = distance along the generator of a cone (see Fig 1)
e = subscript for the outer edge of boundary layer
K = proportionality factor

This note is to show how an expression for K can be obtained in terms of flow parameters for the case in which third-degree polynominals are assumed for the profiles in the boundary layer as follows:[2]

$$u/u_e = (3 - 2\eta)\eta^2 + a\eta(1 - 2\eta + \eta^2) \tag{2}$$

$$w/u_e = (3 - 2\eta)\eta^2(w\ /u\ ) + b\eta(1 - 2\eta + \eta^2) \tag{3}$$

$$(H - H\ )/(H_0 - H\ ) = (3 - 2\eta)\eta^2 + c\eta(1 - 2\eta + \eta^2) \tag{4}$$

where

u = longitudinal velocity
w = circumferential velocity
H = total enthalpy
η = Y/Δ
a,b,c = parameters
s = subscript for surface
0 = subscript for freestream stagnation

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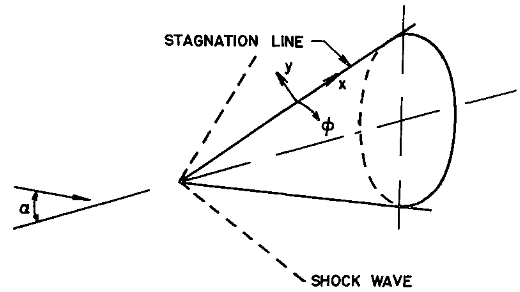

Fig 1 Coordinate system for circular cone at an angle of attack

The momentum equation in circumferential direction when applied at the surface is

$$\frac{dp}{d\phi} = x\beta\left[\frac{\partial}{\partial y}\left(\mu\,\frac{\partial w}{\partial y}\right)\right] \tag{5}$$

where

β = sine of cone semivertex angle
ϕ = circumferential angle (see Fig 1)

Combining Eqs (3) and (5), we have

$$\frac{dp}{d\phi} = xu\ \beta\left\{\frac{\rho/\rho_e}{\Delta^2}\left[b\,\frac{\partial}{\partial\eta}\left(\mu\,\frac{\rho}{\rho}\right) + \mu\left(\frac{\rho}{\rho}\right)\left(6\left(\frac{w_e}{u}\right) - 4b\right)\right]\right\} \tag{6}$$

If the temperature-viscosity relation is assumed to be that of Chapman-Robeson type, i e ,

$$\mu/\mu_e = C\ T/T_e \tag{7}$$

then the first term in the bracket of Eq (6) vanishes, and the expression of $dp/d\phi$ can be simplified as

$$\frac{dp}{d\phi} = x\beta u_e\left\{\left[\frac{\rho/\rho_e}{\Delta^2}\right]\left[\frac{\mu\rho}{\rho_e}\right]\left[6\left(\frac{w_e}{u_e}\right) - 4b\right]\right\} \tag{8}$$

Introducing K in Eq (8) by using Eq (1), we have

$$K^2 = \beta u_e^2\rho\ C\left[\frac{6(w_e/u_e) - 4b}{dp/d\phi}\right] \tag{9}$$

$dp/d\phi$ can be determined from the flow conditions at the outer edge of boundary layer as follows: from the momentum equation of the inviscid flow, the following expression can be derived:

$$\frac{dp}{d\phi} = -\left[\rho\ u^2\left(\frac{w_e}{u_e}\right) + \rho\,u\left(\frac{w}{u}\right)^2\left(\frac{\partial u}{\partial\phi}\right) + \beta\rho\ u^2\left(\frac{w_e}{u_e}\right)\right] \tag{10}$$

Combining Eqs (9) and (10) we have

$$K^2 = \frac{\beta u^2\left[4b - 6\left(\frac{w_e}{u}\right)\right]\left[\frac{\rho\,C_s}{\rho}\right]}{\left[\left(\frac{w}{u}\right)u^2\,\frac{\partial(w_e/u\ )}{\partial\phi} + \left(\frac{w_e}{u}\right)^2 u\left(\frac{\partial u_e}{\partial\phi}\right) + \beta\left(\frac{w_e}{u}\right)u^2\right]} \tag{11}$$

It is thus seen that K can be expressed in terms of the external flow parameters and the profile parameter of the circumferential velocity b

By using Eq (11) and the three boundary-layer equations of a yawed cone, the value of K can be determined with the three profile parameters (a, b, and c) Since the shear